STATISTICS OF THE FRAGMENTS FORMING WITH THE DESTRUCTION OF SOLIDS BY EXPLOSION

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The article sets forth one of the possible approaches to the construction of a size distribution function of the fragments. The Rozin-Rammler law for the distribution is obtained from general theoretical probability consideration. The theoretically obtained distribution function was verified in a large number of experiments. The experimental data are in good agreement with the theoretical deductions.

1. Brittle Failure. If, on explosion, a material is deformed elastically right up to the point of failure, such failure is called brittle. A detailed review on brittle failure may be found in [1].

Here this problem is considered in its most general aspects.

Let us consider cracks existing in a limiting equilibrium. The theory of brittle failure is based on two hypotheses:

1) with the deformation of an elastic-brittle body, there always exists in the body a defect, which is regarded as an isolated crack;

2) the existing crack will be extended if, in this case, there is a decrease in the total potential energy of the system.

A majority of articles on the theory of brittle fractures are devoted to the development of the second hypothesis, in various interpretations. We shall consider some of the results of these investigations, which will be required in the further exposition.

It can be shown [1] that, in the vicinity of the end point of a crack, the stresses σ_x and σ_y approach infinity in accordance with the law

$$\sigma_x = \sigma_y = K / \sqrt{\varepsilon}$$

where ε is a small distance from the tip of the crack. The quantity K is called the coefficient of the intensity of the stresses and is determined by solution of the corresponding problem in the theory of elasticity.

In the theory of brittle failure, the following parameter is introduced

$$K_0 = \left[\frac{E\gamma}{\pi(1-\nu^2)}\right]^{1/2} \tag{1.1}$$

Here E is the Young modulus; ν is the Poisson coefficient; γ is the effective specific energy expended for the formation of a unit of surface of the crack. The quantity γ is the sum of the specific work for the breaking of the interatomic bonds, γ_0, and the specific work of the plastic deformations, γ_p

$$\gamma = \gamma_0 + \gamma_p \tag{1.2}$$

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The crack is in equilibrium, if

$$K = K_0$$

From this, in particular, there is obtained the well known Griffiths formula, which determines the strength, σ_*, of a body containing a crack with a length $2l_*$

$$\sigma_* = \left[\frac{E\gamma}{\pi(1-\nu^2)l_*}\right]^{1/2} \tag{1.3}$$

It is essential for the further exposition that the elastic-brittle properties of materials, in the given statement of the problem, are characterized by a single parameter, K, with the dimensionality $ML^{1/2}T^{-2}$.

For application of the theory of brittle fractures to the problem of explosive destruction, the problem of the static interaction of the system of cracks is essential. The simplest of such systems in an infinite number of parallel cracks, disposed symmetrically with respect to the y axis, loaded from within by a constant pressure p. This problem has been solved by many authors [1]. The solution is obtained in the form of series, or approximately in finite analytical form [3]

$$K = p\left(\frac{h}{2\pi}\,\mathrm{th}\,\frac{\pi l}{h}\right)^{1/2} \tag{1.4}$$

where h is the distance between cracks; l is the half-length of a crack.

The theory of equilibrium brittle fractures does not consider the process of the generation of the cracks. Therefore, in formula (1.4) there are two unknown parameters, h and l. As a hypothesis, the value of l may be fixed using the relationship (1.3), which takes account of the value of the defect of the characteristic of the given material. Then (1.4) determines the distance between cracks with stresses σ, exceeding the strength of the material.

Formulas (1.3) and (1.4) can be transformed to a more convenient form. If, in the Griffiths formula (1.3), we set $l_* = b \approx 10^{-8}$ cm (the interatomic distance), it determines the so-called theoretical strength, σ_0. In this case, it is necessary to set $\gamma = \gamma_0 = {}^1\!/_2\sigma_0 b$. It is then easy to obtain

$$\sigma_0 = \frac{E}{\pi(1-\nu^2)} \approx 0.1\,E, \quad l_* = \frac{b(\gamma_0+\gamma_p)}{\gamma_0}\left(\frac{\sigma_0}{\sigma_*}\right)^2$$

and from (1.4)

$$\frac{h}{\pi l_*}\,\mathrm{th}\,\frac{\pi l_*}{h} = \left(\frac{\sigma_*}{p}\right)^2 \quad \text{at} \quad l = l_*,\ K = K_0 \tag{1.5}$$

From the results obtained in the dynamics of brittle failure, we single out two.

1. The velocity of the cracks can not exceed some limiting value. This result has been obtained in a number of theoretical and experimental articles, references to which may be found in [1]. Theoretically, the limiting velocity of the development of the cracks is equal to the Rayleigh velocity. Its experimentally found values are half of this quantity.

2. The state of stress in the neighborhood of the tip of a moving crack differs only slightly from the state of stress of a motionless crack, with the same geometry and identical external forces.

This result was obtained experimentally in [2]. On the basis of these two results, the following formula may be constructed for determining the velocity of the motion of the crack [3]:

$$C = C_*\sqrt{1 - K_0/K} \tag{1.6}$$

Here C_* is the limiting velocity of the crack; K is the coefficient of the intensity of the stresses; K_0 is its equilibrium value.

An experimental verification of this formula is given in [4]. We now consider the following problem. In the plane xy, let there exist an infinite system of parallel cracks with a length of $2l$, disposed symmetrically with respect to the y axis, at a distance h_0 apart. At the initial moment of time, there is built up within the cracks a pressure, p, which exceeds the equilibrium pressure, and which remains constant during the whole time of the motion. It is required to determine the movement of the cracks and, in particular, to investigate the stability of this movement. The solution of this problem is given in [3]. The rate of development of the cracks is determined at once using formulas (1.4) and (1.6). Such a system of cracks has an instability of the following type. Let all the cracks receive an identical increment of length. Then, the velocity of the cracks with a large length increases, while the velocity of the smaller cracks decreases until they stop.

It has been demonstrated that, if the length of the large cracks exceeds by e times the length of the small cracks, the state of stress in the neighborhood of the large cracks does not depend on the presence of the small cracks. A new system of cracks is formed with a distance of $2h_0$ between them, with which the same procedure can be carried through. Thus, with the passage of time, the distance between cracks has a tendency to increase.

If the cracks traverse a distance L, the number of possible acts of doubling, N, is equal to $\ln(L/l)$, and the distance between the cracks is

$$h = h_0 2^N = h_0 (L / l_0)^{\ln 2} \qquad (1.7)$$

2. Simple Size-Distribution Function of Fragments. The probability character of the development of cracks and the development of fragments is the basis of the theory of brittle failure.

In fact, if we return to the first of the hypotheses on which the Griffiths-Erwin theory is based, the following is evident: with the deformation of an elastic-brittle body, in the body there are an arbitrary number of defects, regarded as cracks.

The strength of a material with a static load is determined by the behavior of a single crack, evidently the largest. With a dynamic load and, in particular, with an explosion in an arbitrary finite volume of a solid, there develops simultaneously a large number of fractures, leading to the formation of fragments of the most varied dimensions, volumes, and forms. It is clear from the foregoing that an adequate description of the fragmentation (crushing) action of an explosion must be based on theoretical probability concepts. The definite schemes of failure which have been discussed in the preceding section, must determine the linear dimension of a fragment, understood in the "mean" sense.

From the same starting point, it is postulated that each fragment has some characteristic linear dimension, i.e., fragments of the "needle" type are not considered.

In the statistical analysis of the formation of fragments, use is sometimes made of a normal law of distribution with respect to the particle size, or to the logarithms of the sizes [5], as well as of the Poisson law [6]. In addition to these laws, obtained on the basis of theoretical probability considerations, there exist also a number of empirical relationships, used mainly in the ore-beneficiation industry. Among these, there must be noted the Rozin-Rammler law

$$V(x) = V_0 e^{-ax^n} \qquad (2.1)$$

Here V_0 is the total volume of the mass of broken-down material being considered; x is the characteristic dimension of a fragment; V(x) is the volume of all the fragments whose dimension exceeds x; a and n are empirical parameters.

An analysis of the results of industrial explosions [7], and of the experiments carried out in the present work, shows that this relationship may be applied with a sufficient degree of accuracy to the analysis of an exploded mass, as well as of fragments formed with the breakdown of some of the simplest constructions.

We shall show that the Rozin-Rammler law is obtained as a partial case of general probability concepts, having a definite physical meaning.

Let the distribution function (the probability that the fragments will have a linear dimension less than some given value of x) have the form

$$\Phi(x) = 1 - e^{-F(x)}, \qquad F(0) = 0, \quad F(\infty) = \infty \qquad (2.2)$$

Here F(x) is a positively determined function, whose derivative may have a finite number of discontinuities of the first kind over the whole interval of the change in x.

The probability that the fragments will have a length in the range (x, x + dx) is determined as

$$dp = \Phi'(x)\,dx = F'(x)\,e^{-F(x)} \tag{2.3}$$

The number of fragments in this range is

$$dm = \frac{V_0}{v}\,dp = \frac{V_0}{v}\,F'(x)\,e^{-F(x)}dx \tag{2.4}$$

Here ν is some mean volume of a particle having dimensions within the range (x, x + dx).

Formula (2.4) clarifies the concept of probability. In the given case

$$dp = \frac{v}{V_0}\,dm = \frac{dV}{V_0}, \qquad \Delta p_i = \frac{\Delta V_i}{V_0} \tag{2.5}$$

Thus, the probability that a particle will have a dimension lying within the range of values from x to x + Δx is the ratio of the volume of all the particles having the given dimensions to the total volume of the mass being analyzed. The volume of all the particles whose dimensions are greater is determined from (2.3) and (2.4)

$$V(x) = \int_x^\infty v\,dm = V_0 e^{-F(x)} \tag{2.6}$$

The Rozin-Rammler law is obtained from this, if we set $F(x) = ax^n$. The normalizing condition

$$\int_0^\infty dp = 1$$

which is satisfied as a result of the limitations imposed on F(x), determines the degree of accuracy of the given approach. Since, in reality, the dimension of the fragments varies not from 0 to ∞, but from some minimal dimension x_{min} to a maximal dimension x_{max}, the accuracy must be determined by satisfaction of the inequality

$$\int_0^{x_{min}} dp \ll 1, \qquad \int_{x_{max}}^\infty dp \ll 1$$

We shall show that it is possible to determine F(x), starting from theoretical probability concepts. We formulate the hypotheses.

1. It is postulated that all the faces of the fragments are planar, and that there are always two parallel faces. This postulation permits a "multidimensional" problem for the formation of a fragment to a "one-dimensional" problem, and makes it possible to consider merely the process of the development of two plane fractures, located at a certain distance, x, one from the other.

Since the appearance of a fracture at a given point is accompanied by the unloading of the material in the neighborhood of this point, it is evident that the specific probability $P_1(dx/x)$ of the development of a second crack at a distance x from the first, in the range of dx, depends on the value of x. In addition, $P_1(dx/x)$ is proportional to dx

$$P_1(dx\,/\,x) = \varphi(x)\,dx$$

If a homogeneous material is being considered, it is clear that the greater the distance x from the first crack, the more probable the appearance of a second crack.

2. For a given state of stress, there exists a characteristic linear dimension x_0, which is such that the probability of the development of a second crack at a distance x_0 is greater than at a distance $x < x_0$. This postulation can be written, for example, in the form

$$P_1\left(\frac{dx}{x}\right) = \frac{n}{x_0}\left(\frac{x}{x_0}\right)^{n-1}dx \tag{2.7}$$

where x_0 and n are some parameters, and $n > 1$. If $n = 1$, (2.7) leads to a basic relationship, which occurs in the derivation of the Poisson law for the distribution of points around a straight line. In this case, the probability that a point will fall within the segment dx does not depend on the presence of points in the adjacent segments.

3. The probability of the development of two cracks in an infinitely small segment dx is equal to zero:

$$P_2(dx) = 0, \qquad P_0(dx) = 1 - P_1(dx) \tag{2.8}$$

Let us calculate the probability that there are no cracks in the segment (x + dx). In accordance with the formula for the multiplication of probabilities:

$$P_0(x + dx) = P_0(x)\, P_0(x^{-1}dx) \tag{2.9}$$

Substituting here (2.7) and (2.8), we obtain

$$P_0(x + dx) = P_0(x)\left[1 - \frac{n}{x_0}\left(\frac{x}{x_0}\right)^{n-1} dx\right]$$

From this

$$\frac{dP_0}{dx} = -P_0 \frac{n}{x_0}\left(\frac{x}{x_0}\right)^{n-1}, \qquad P_0 = \text{const} \cdot \exp\left[-\left(\frac{x}{x_0}\right)^n\right]$$

The constant is determined from the condition $P_0(0) = 1$. We thus obtain

$$P_0(x) = \exp\left[-\left(\frac{x}{x_0}\right)^n\right]$$

It is evident that $P_0(x)$ is the probability that a fragment will have a length greater than x, i.e., the distribution function (2.2) is connected with P_0 by the relationship

$$\Phi(x) = 1 - P_0(x)$$

The differential probability, dp, is the probability of the simultaneous occurence of two events: no cracks in the segment x, and one crack in the segment dx,

$$dp = P_0(x)\, P_1(x^{-1}dx) = \frac{n}{x_0}\left(\frac{x}{x_0}\right)^{n-1} \exp\left[-\left(\frac{x}{x_0}\right)^n\right] dx \tag{2.10}$$

This expression naturally coincides with (2.3) if, in the latter, we set

$$F(x) = (x/x_0)^n$$

Substituting this expression into (2.6), we obtain the Rozin-Rammler law in the form

$$V(x) = V_0 \exp\left[-(x/x_0)^n\right] \tag{2.11}$$

The mean size of the fragments is calculated by the usual method

$$\langle x \rangle = \int_0^\infty x\, dp \tag{2.12}$$

hence, after substitution of (2.10) and the calculations, we have

$$\langle x \rangle = x_0 \Gamma(1 + 1/n), \qquad \Gamma(1 + 1/n) = \int_0^\infty e^{-t} t^{1/n} dt \tag{2.13}$$

The dispersion of the value of x is calculated by the usual method using the formula

$$D = \int_0^\infty (x - \langle x \rangle)^2 \, dp = \langle x \rangle^2 \left[\frac{1 + 1/n}{\Gamma(1 + 1/n)} - 1 \right] \tag{2.14}$$

As noted above, in the case under consideration, $n \gg 1$, so that the argument of the Γ-function in (2.13) and (2.14) varies within the range from unity to two. At those values of the argument, the Γ-function has a value on the order of unity. We have the approximate relationships

$$\langle x \rangle \approx x_0, \qquad D \approx \langle x \rangle^2 / n \tag{2.15}$$

which bring out the statistical meaning of the parameters n and x_0 in the Rozin-Rammler law, in the form (2.11). In this case, as a result of (2.5), the mean size of a fragment is understood as the "mean-suspended" value. Experimental values will be denoted by the subscript z. The experimental determination of the mean value is carried out using the formula

$$\langle x_z \rangle = \sum_{i=1}^{j} x_{iz} \Delta p_i = \sum_{i=1}^{j} x_{iz} \frac{\Delta V_{iz}}{V_0} \tag{2.16}$$

Here x_{iz} is the mean size of the i-th group; ΔV_i is the volume (weight) of the i-th group; V_0 is the total volume (weight) of the mass being analyzed; j is the number of groups.

The parameter n determines the uniformity of the pulverization. It is evident that the lower the value of the dispersion, the more "clustered" will be the arrangement of all the values of x with respect to the point $x = \langle x \rangle$, i.e., the more uniform will be the pulverization. It follows from (2.15) that the uniformity of the pulverization increases with a rise in the value of n. The overexpenditure of power for the regrinding of ore is bound up with the same parameter.

We assume that all the fragments forming after an explosion are geometrically similar. Then, a fragment, having the characteristic dimension x, has the surface s and the volume v, equal, respectively, to

$$s = k_s x^2, \qquad v = k_v x^3$$

where k_s and k_v are constant coefficients. The total surface of all the fragments is determined by the expression

$$S = \int_0^\infty s \, dm = V_0 \frac{k_s}{k_v} \int_0^\infty \frac{dp}{x} = \frac{V_0}{x_0} \frac{k_s}{k_v} \Gamma\left(1 - \frac{1}{n}\right)$$

The optimal variant of the pulverization is achieved when the total volume is broken up into identical pieces with a size $\langle x \rangle$. Under these circumstances, the surface of all the fragments is

$$\langle S \rangle = \frac{V_0}{\langle x \rangle} \frac{k_s}{k_v}$$

The relative fraction of "excess" surface is

$$\frac{\Delta S}{S} = \frac{S - \langle S \rangle}{S} = \frac{x}{x_0} \Gamma\left(1 - \frac{1}{n}\right) = \Gamma\left(1 + \frac{1}{n}\right) \Gamma\left(1 - \frac{1}{n}\right)$$

In this sense, the case $n = 1$, corresponding to a Poisson distribution, is the most disadvantageous; the total surface of all the particles and, consequently, the energy expended for its formation, approaches infinity. With an increase in the value of n, this value decreases rapidly and, with $n = 1.5$, is 1.42. We note in conclusion that, at large values of n, (on the order of 2-3), the distribution law (2.10) is very close to a normal (Gaussian) distribution.

3. Experimental Verification of Simple Distribution Function. For the one-dimensional failure model proposed above, the best experimental approximation is obtained by the explosive destruction of rings. For the experiments, these rings were made of aluminum.

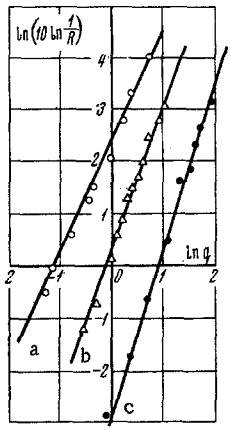

Fig. 1

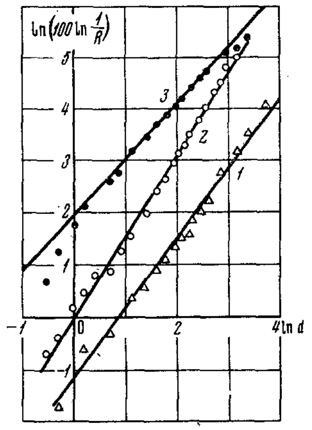

Fig. 2

With the parameters of the explosives and of the material of the rings selected in a corresponding manner, the main mass of the fragments was formed by radial cracks. Collection of the fragments after a blast presents considerable experimental difficulties. To achieve a sufficiently convincing analysis of the fragments, as large an amount of them as possible must be caught. In addition, the fragments must be braked as slowly as possible, to avoid their secondary pulverization. In the experiments carried out, the fragments were braked using snow. The experiments were carried out in a sufficiently large chamber, filled with packed snow. The charge of explosive (TG 50/50), in the form of a cylinder, was placed inside the ring on a paper base, and was located at the center of the chamber.

After the explosion, the chamber was washed with hot water, and the fragments were easily removed from it. Thus, fragments with a total weight from 90 to 100% of the original weight of the ring were successfully collected. In each series, with fixed parameters of the explosive and of the ring, from 3 to 5 experiments were made, depending on the amount of fragments formed. The total number of fragments in each series was from 50 to several hundred pieces. The experimental results were set up in the form of a table of values of $R(x) = V(x)/V_0$, corresponding to different values of x. In the case of rings, the value of x was measured by the weight of a fragment. Then, curves of R(x) were plotted on a log-log scale. In accordance with (2.11), in the coordinates ln ln(1/R), ln x, this dependence should be expressed by the straight line

$$\ln \ln (1 / R) = n (\ln x - \ln x_0) \tag{3.1}$$

Examples of such curves are given in Figs. 1-4. Figure 1 gives the results of the destruction of Duralumin rings. The rings had a constant diameter of 80 mm, a constant height of 10 mm, and thicknesses of 2,4, and 8 mm, which, on Fig. 1, correspond to curves a, b, and c. The charge of explosive was in the form of a cylinder of TG with a weight of 18 g, a diameter of 35 mm, and a height of 10 mm; the values of the parameters n and x_0 are given in Table 1. As is evident, the experimental points are well described by formula (3.1).

The second group of experiments was carried out using a ring of constant thickness (2 mm), but with charges of different weights: Q = 4, 8, 18, and 43 g. The distribution parameters for this group of experiments are also given in Table 1.

The mean values of $\langle x_z \rangle$, shown in Table 1, were calculated using formula (2.13). These values practically coincide with the mean values calculated using formula (2.5). The divergence between the two cases does not exceed 10%.

The 4th, 5th, and 6th columns of Table 1 may be used to establish the dependence of x_0 on the weight of the charge Q in the form

$$x_0 = \text{const}\, Q^{-2/3} \tag{3.2}$$

The fourth point shows a deviation toward the side of a decrease in x_0. Up to the point where the diameter of the charge is small in comparison with the diameter of the ring, it can be assumed that the pressure at the front of the shock wave is proportional to the cube root of the weight of the explosive. The distension forces arising in the ring evidently have a value directly proportional to the pressure in the shock wave. Denoting the distension stress by p, we have

$$x_0 \sim p^{-2}$$

This result is in agreement with formula (1.5), in which it is necessary to set $h = x_0$; since $\text{th}(\pi l/h) \approx 1$ at $l \geq h$, from (1.5) we have

$$x_0 = \pi l_* (\sigma_* / p)^2$$

TABLE 1

	δ, min			Q, g			
	2	4	8	4	8	18	43
n	2.2	2.8	3.3	1.9	2.7	2.2	1.3
x_0, cm	1.7	1.7	2.0	4.7	2.7	1.7	0.5
$\langle x \rangle$, cm	1.4	1.4	1.8	4.1	2.4	1.5	0.4

In accordance with the data of Table 1, the dependence of the mean diameter on the thickness of the ring, δ, with a constant weight of explosive, can be represented in the form $x_0 \sim \delta^{0.3}$. Qualitatively, this result is in agreement with formula (1.7); the divergence in the power exponents can be explained by the high degree of simplification of the model problem.

To clarify the distribution function in the more general case, laboratory tests were made with samples of more complex construction. The explosions were carried out in cylindrical blocks, with a height equal to their diameter, made of Plexiglas as well as of Mendeleev cement. A concentrated charge (hexogene) was placed at the center of the block. The procedure used in collecting the fragments was the same as in the case of the metallic rings. As a result of the great number of fragments formed (in some experiments, on the order of 10^6), the particle size analysis was done by screening through a sieve with calibrated openings from 0 to 24 mm.

The results of three series of experiments in Plexiglas blocks with a size of 140 mm, and with charges weighing 3, 10, and 100 g, are shown graphically in Fig. 2 (straight lines 1, 2, and 3, respectively). As is evident, in this case the Rozin-Rammler distribution function describes the experimental data well.

The fact that distribution function (2.11) describes not too badly the results of laboratory tests with cylindrical blocks suggests the application of the Rozin-Rammler law to the analysis of the particle-size composition of an exploded mass, and to more complex cases. It is to be expected that the accuracy of such an analysis will be lower than in the preceding cases.

4. Distribution Function of Fragments with the Explosion of Structures of Arbitrary Form. Let us consider the action of a concentrated charge in a continuous or bounded mass of rock. The physical picture of the breakdown is approximately the following. Compression and expansion waves, passing through the medium, bring about the opening up to microcracks, whose propagation with the subsequent motion of the medium under the action of the detonation products leads to the formation of fragments.

Since the intensity of the waves decreases with increasing distance from the charge, there is also a decrease in the stresses which lead to breakdown.

We shall consider the formation of fragments in each elementary spherical layer of radius r and thickness dr. It is evident that the mean dimensions of the fragments in each layer increase with increasing distance from the center of the blast. We make the following assumptions:

1) in each elementary layer, the distribution of the fragments is described by the functions (2.11)

$$V_r(x) = 4\pi r^2 \exp\left[-(x/x_0)^n\right] dr$$

2) the value of n does not vary with distance, while x_0 increases in accordance with the law

$$x_0 = Ar^{\omega}$$

The volume of all the fragments having a dimension greater than x is equal to

$$V(x) = \int_{r_0}^{r} 4\pi r^2 \exp\left[-\left(\frac{x}{x_0(r)}\right)^n\right] dr \tag{4.1}$$

The integration is carried out within the limits from the radius of the cavity to the breakdown radius R_0, in the case of an unbounded mass. If the blast takes place in a bounded region, by R_0 there must be understood its characteristic dimension. In all cases which are important in practice, $R_0 \gg r_0$.

We integrate (4.1) by parts

$$V(x) = \frac{4\pi}{3}\left\{R_0^3 \exp\left[-\left(\frac{x}{x_0(R_0)}\right)^n\right] - r_0^3 \exp\left[-\left(\frac{x}{x_0(r_0)}\right)^n\right]\right\} - \frac{4\pi\omega n}{3}\int_{r_0}^{R_0}\left(\frac{x}{x_0}\right)^n \exp\left[-\left(\frac{x}{x_0}\right)^n\right] r^2 dr \tag{4.2}$$

TABLE 2

Expt. No.	Q	Q°	x_0	n
1	0.5	538	6.36	1.92
2	0.5	4866	19.3	2.04
3	0.2	2353	26.6	1.73
4	0.2	1250	13.7	2.05
5	0.1	1727	23.3	1.8
6	0.1	1221	23.3	1.77
7	0.04	855	55	1.32
8	0.04	1388	52	1.52
9	0.02	859	39.6	1.68
10	0.0005	2.13	4.7	1.29
11	0.0005	0.8	1.5	1.28

TABLE 3

Expt. No.	Q	Q°	x_0	H	n
1	0.5	1711	28.2	87	1.59
2	0.5	1862	19.9	70	1.3
3	0.2	465	14.6	48	1.7
4	0.1	500	20.7	50	1.56
5	0.1	220	13.5	40	1.5
6	0.05	161	15.6	30	1.44
7	0.05	332	11.2	30	1.47
8	0.02	79	8	20	0.91
9	0.02	29	31.2	20	1.32

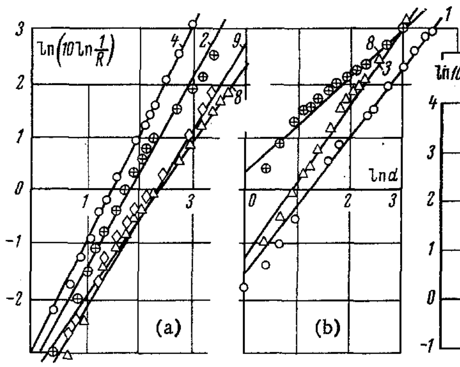

Fig. 3

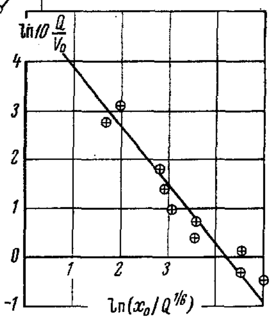

Fig. 4

An evaluation shows that the value of the second term in the right-hand part of this equality may be neglected in comparison with the first term, if the condition $\omega n > 2$ is fulfilled. Since the n is everywhere ≥ 1, this term may be neglected if $\alpha > 2$.

We postulate that this inequality holds. Then, from (4.2), taking into account that $R_0 \gg r_0$, we obtain

$$V(x) = V_0 \exp\left[-\left(\frac{x}{x_0(R_0)}\right)^n\right], \quad V_0 = \frac{4\pi R_0^3}{3} \tag{4.3}$$

Thus, in this case also, we go over to the Rozin-Rammler formula.

As is evident, the mean size of a fragment is determined by the state of stress at the boundary of the breakdown zone with a blast in an unbounded medium, or at the bounding surface of a mass. This result must be understood to be purely relative since, in the first case, there is no clearly defined radius of the breakdown zone and, in the second case, the state of stress near a free boundary is, generally speaking, unknown. By the boundary of the breakdown zone, there is usually understood a surface at which some static criterion of breakdown is satisfied, for example, equality of the distension stress and the tensile strength. In the theory of brittle failure, this means that there is one limiting fracture, whose length is determined by expression (1.3). It is clear that for the statistical analysis of fragments, when the presence of a large number of cracks is understood, such a determination is not feasible. Nevertheless, expression (4.3) may be found useful to construct an empirical formula, for the purpose of decreasing the number of parameters to be determined experimentally.

We postulate that the distension stress arising in a medium with the explosion of a charge of weight Q at a distance r from the center of the blast, is determined as

$$\sigma_r = B_0 (Q^{1/3}/r)^{\xi} \quad (B_1 = \text{const}) \tag{4.4}$$

Here, the constant depends on the properties of the explosive and of the medium. We assume further that the brittle properties of the medium are determined by the parameter K_0 (1.1). Then, from considerations of dimensionality, we have

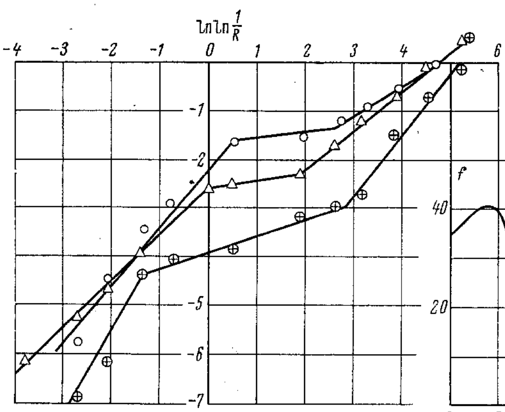

Fig. 5

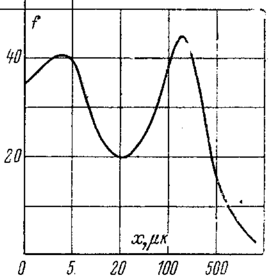

Fig. 6

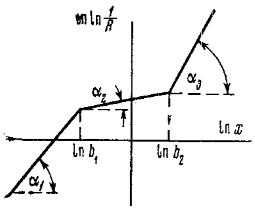

Fig. 7

$$x_0 = B_2 \frac{K_0^2}{\sigma_r^2} \qquad (B_2 = \text{const}) \tag{4.5}$$

From these expressions, at $r = R_0$, we obtain

$$x_0 = B_3 K_0^2 \left(\frac{R_0}{Q^{1/3}}\right)^{2\xi}$$

or, introducing the specific consumption of explosive, $w = Q/V_0$, we have

$$x_0 = B_4 K_0^2 (V_0/Q)^{2/3\xi} = B_5 K_0^2 w_4^{-2/3\xi} \qquad (B_4, B_5 = \text{const}) \tag{4.6}$$

In this expression, x_0 does not depend on the scale of the explosion which, in the general case, corresponds to the experimental data. It is a question of the fact that the considerations leading to formula (4.6) are essentially based on statistical considerations. It is found that the scale factor is directly connected with the kinetics of the fractures. We assume that there are being considered two explosions of spherical charges, Q and k^3Q, in an unbounded mass, or in geometrically similar pieces. Then, at similar distances, i.e., r in the first case and kr in the second, the stresses arising with the passage of a wave, at corresponding moments of time, will clearly be identical. In accordance with statistical considerations, the systems of cracks originally created will be the same.

However, the time of action of these stresses is greater in the second case. If it is roughly assumed that the wave length in the second case is k times greater than in the first case, and that development of the cracks occurs in a definite constant part of the wave length, we obtain from this that the development of a network of cracks in the second case develops over a period of time which is k times greater than in the first case. It was demonstrated at the end of Section 1 that the development of a system of cracks is of an unstable nature: if some crack, by chance, becomes larger than the adjacent cracks, its velocity increases, while that of the adjacent cracks decreases. For the simplest system of cracks, the distance between them with the passage of a wave length L increases in the ratio $(L/l_0)^{\ln 2}$, where $2l_0$ is the original length of the cracks.

In the general case, the power exponent may be different, since the original network of cracks is of a more complex structure.

If, by L, we understand a wave length proportional to $Q^{1/3}$, then, introducing an additional factor into (4.6), we obtain the empirical formula

$$x_0 = B (V_0/Q)^{2/3\xi} Q^{1/3\beta} \tag{4.7}$$

Here β is a scale factor; ξ is the attenuation coefficient of the stresses. At this stage of the investigation it is hardly possible to assign a concrete meaning to the constant B on the basis of formulas (4.4) and (4.5), in view of their rough approximate nature. In addition, the value of K_0, for example, for a majority

TABLE 4

№	n_1	n_2	n_3	$\ln b_1$	$\ln b_2$	$\langle x \rangle$ (mm)	$\langle x_z \rangle$ (mm)
1	1.26	0.2	0.7	0.5	2.6	132	112.9
2	1.05	1.125	0.78	0	1.9	169	125.9
3	1.7	0.3	1	1.4	2.8	193	183.2

of rocks is simply unknown since, in accordance with formulas (1.1) and (1.2), it includes the specific work of plastic deformations. Thus, in formula (4.7), the parameters B, ξ, and β remain to be determined.

5. Experimental Explosions in Rocks. An experimental verification of the applicability of the Rozin–Rammler law in the statistic analysis of the particle-size composition of an exploded mass of rock was first carried out in [7].

On the basis of an analysis of a large number of industrial explosions it has been demonstrated that formula (2.1) describes the experimental data sufficiently well. However, in calculation of the mean size of a piece, the authors of [7] did not obtain satisfactory results. The divergence between the theoretical and experimental values was 50–60%. To make this situation more precise, the present authors have carried out additional experiments.

The experiments were carried out on limestone, of the eighth strength category. Two series of experiments were made. In one of them, the explosions were carried out on individual rocks, and in the other in an outcropping of a continuous mass. Charges of hexagene were used. The weight of the charge was varied in the range from 20 to 500 g. Each experiment was repeated 2-3 times with exactly the same weight of explosive and approximately identical sizes of the rocks. The particle-size composition of the exploded mass was analyzed using a sieve-type screen, and the corresponding fractions were weighed. The ratio of this weight to the total weight of the fragments collected determines the quantity

$$R(x) = V(x) / V_0$$

In the given case, the parameter x is the diameter of the opening in the corresponding sieve, d. The parameters of the explosions and the analytical results are given in Tables 2 and 3, as well as in Fig. 3a and b, respectively, for the explosions in individual rocks and in the outcropping.

In the presence of considerable scatter, straight lines were passed through the experimental points, using the method of least squares. The number of the straight line on the figures corresponds to the number of the experiment. In addition, the mean value of $\langle x_Z \rangle$ was determined, calculated directly from experiments, using formula (2.16).

The maximal divergence between the values of $\langle x \rangle$ and $\langle x_Z \rangle$, calculated using formula (2.13), was not more than 15% and, in a majority of cases, not more than 4–6%.

Let us compare the data for two experiments in rocks (Table 2). A charge with a weight of 500 g in a rock with a weight of 5000 kg, gives a mean fragment diameter of $\approx$ 19 cm.

With the explosion of a charge with a weight of 0.5 g in a rock with a weight of 2.130 kg, there are formed fragments with a mean size of $\approx$ 50 cm. With an increase of the scale by approximately 10 times, the dimension of a fragment increases by four times. It may be assumed that, with an increase in the scale of the explosion by k times, the mean dimension of a fragment increases by $k^{1/2}$. Thus, in formula (4.7), $\beta = 1/2$. The dependence of the mean dimension of a fragment on the specific consumption of explosive can be obtained if all the data of the tables are plotted on a curve in the coordinates $\ln(10\, Q/V_0)$, $\ln(x_0/Q^{1/6})$. This curve is shown on Fig. 4, and the final result can be represented in the form of the equation

$$x_0 \approx 10 Q^{1/6} (V_0 / Q)^{4/5} \tag{5.1}$$

Here Q is the weight of explosive, kg; V is the volume of soil broken up, m^3; x_0 is the mean size of a piece, cm.

With explosions in an outcropping, a formula analogous to (5.1) has not been obtained up to the present time, since the range of change in x_0 in the experiments was very small.

6. Effect of a Nonhomogeneous Medium on the Structure of the Distribution Function. If the material being broken up by an explosion contains nonhomogeneities and previously perturbed microscopic cracks, the size distribution of the fragments becomes more complex. As an example, let us consider the results of one of the experiments presented in [8]. The samples exploded consisted of sandstone which, as is well

known, is made up of extremely strong grains, disposed in a less strong cementing mass. An analysis of the data of [8], in the coordinates $\ln \ln R^{-1}$ and $\ln x$, leads to the curve shown in Fig. 5. If, as in [8], we plot graphically the dependence of the density of the distribution on the sizes of the fractions, we obtain a curve with two maxima (Fig. 6). One of these corresponds to the mean size of the strong grains, and the second to the mean size of the fragments formed as the result of external action.

Formally, we are here considering two associated distributions: one given by the structure of the medium, the other the result of explosive action. The previously described approach to the determination of the particle-size composition of an exploded mass can be extended also to this more complex case.

Let us consider the distribution function $1-R(x)$ in the form (2.2), where $F(x)$ is an arbitrary function.

For a case similar to that illustrated in Fig. 5, where the curve of the dependence of $\ln \ln R^{-1}$ on $\ln x$ is represented by three straight lines, the function $F(x)$ may be represented in the form

$$F(x) = \exp\left(a + \frac{n_1 + n_3}{2} \ln x + \frac{n_2 - n_1}{2} |\ln b_1 x| + \frac{n_3 - n_2}{2} \left|\ln \frac{x}{b_2}\right|\right)$$

where a, b_1, b_2, n_1 n_2, n_3 are the distribution parameters. Such a scheme is shown schematically in Fig. 7, which shows also the designation of the characteristic points in terms of the distribution parameters. Substituting $F(x)$ into (2.2) and taking double logarithms, we obtain

$$\ln \ln R^{-1} = \begin{cases} n_1 \ln x + C_1 & \text{at} \quad \ln x \leqslant -\ln b_1 \\ n_2 \ln x + C_2 & \text{at} \quad -\ln b_1 \leqslant \ln x \leqslant \ln b_2 \\ n_3 \ln x + C_3 & \text{at} \quad \ln x \geqslant \ln b_2 \end{cases}$$
$$C_1,\ C_2,\ C_3 = \text{const}$$

For the differential probability, dp, we have

$$dp = F'(x)\, e^{-F(x)} dx = f(x)\, dx$$

The function $F(x)$ is determined by the following expressions:

$$F(x) = e^a b_1^{(n_1-n_2)/2} b_2^{(n_3-n_2)/2} x^{n_1} \quad \text{at} \quad 0 \leqslant x \leqslant 1/b_1$$
$$F(x) = e^a b_1^{(n_2-n_2)/2} b_2^{(n_3-n_2)/2} x^{n_2} \quad \text{at} \quad 1/b_1 < x \leqslant b_2$$
$$F(x) = e^a b_1^{(n_2-n_1)/2} b_2^{(n_2-n_3)/2} x^{n_3} \quad \text{at} \quad b_2 < x < \infty$$

In accordance with this, the distribution density is

$$f(x) = \frac{n_i}{x_{0i}} \left(\frac{x}{x_{0i}}\right)^{n_i - 1} \exp\left[-\left(\frac{x}{x_{0i}}\right)^{n_i}\right] \quad (i = 1, 2, 3)$$

where the subscripts 1, 2, 3 correspond to the intervals of change in x: $(0, 1/b_1)$, $(1/b_1, b_2)$, (b_2, ∞).

In this case, the values of x_{0i} are the following:

$$x_{01} = [e^{-a} b_1^{(n_2-n_1)/2} b_2^{(n_2-n_3)/2}]^{1/n_1}$$
$$x_{02} = [e^{-a} b_1^{(n_1-n_2)/2} b_2^{(n_2-n_3)/2}]^{1/n_2}$$
$$x_{03} = [e^{-a} b_1^{(n_1-n_2)/2} b_2^{(n_3-n_1)/2}]^{1/n_3}$$

In the given case, the mean size of a fragment, calculated using (2.12), is represented in the form of the following formula:

$$\langle x \rangle = x_{01} G_1 \left[\left(\frac{1}{b_1 x_{01}}\right)^{n_1}, 1 + \frac{1}{n_1}\right] + x_{02} \left\{G_1\left[\left(\frac{b_2}{x_{02}}\right)^{n_2}, 1 + \frac{1}{n_1}\right]\right.$$
$$\left. - G_1\left[\left(\frac{1}{b_1 x_{02}}\right)^{n_2}, 1 + \frac{1}{n_2}\right]\right\} + x_{03} G_2 \left[\left(\frac{b_2}{x_{03}}\right)^{n_3}, 1 + \frac{1}{n_3}\right] \tag{6.1}$$

Here $G_1(x, a)$ and $G_2(x, a)$ are incomplete Γ-functions

$$G_1(x, a) = \int_0^x e^{-t} t^{a-1} dt, \quad G_2(x, a) = \int_x^\infty e^{-t} t^{a-1} dt$$

Let us consider the results of three experiments, taken from [8]. These data are used to plot the curves shown in Fig. 5. The values of the parameters b_1, b_2, n_1, n_2, n_3, ln b_1, and ln b_2, determined using these curves, as well as the mean values of $\langle x \rangle$, calculated using formula (6.1) and of $\langle x_Z \rangle$, determined directly from experimental data, are given in Table 4. As is evident from the curves presented, a generalized Rozin-Rammler law, in the form (2.2), can be applied with sufficient accuracy to the analysis of the particle-size composition of an exploded mass and to calculation of the mean size of a fragment. We note that fractions owing their origin to the structural inhomogeneity of the material are described by the right-hand part of the curves given in Fig. 5. In this case, as is evident from Table 4, n_3 is always less than unity. For the breakdown of homogeneous materials, the distribution of the fragments is described by formula (3.1) with $n > 1$. This fact was essential in the construction of the simple distribution function. In the case under consideration, the value of n_3 must be less than unity; this follows from the following considerations. Let us assume that the object being broken down consists of very strong blocks with a mean size of x_{03}, cemented together by a less strong material. It is clear that the formation of a fragment with a size less than x_{03} will be less probable than the formation of a fragment containing several strong blocks. In other words, in the given case, the exponent in formula (3.2) must be negative, or $n < 1$. In the case when the inhomogeneity of the medium is due to its highly developed fissility, the situation will be the opposite. The probability of the formation of a fragment with a size greater than that determined by a system with previously developed cracks is less than that of a fragment of smaller size.

In this case we have two straight lines with the slopes tg $\alpha_1 = n_1 > 1$, tg $\alpha_3 = n_3 > 1$, connected by a segment with the slope tg $\alpha_2 = n_2 \gtreqless 1$. Generally speaking, the question of the effect of previous fissility has, at the present time, been insufficiently investigated. The main reason for this is that there is no satisfactory method for the experimental determination of fissility within a mass of rock.

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